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Cellular automata models of traffic flow along a highway containing a junction

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Abstract. We examine various realistic generalizations of the basic cellular automaton model describing traffic flow along a highway. In particular, we introduce a *slow-to-start* rule which simulates a possible delay before a car pulls away from being stationary. Having discussed the case of a bare highway, we then consider the presence of a junction. We study the effects of acceleration, disorder, and slow-to-start behaviour on the queue length at the entrance to the highway. Interestingly, the junction's efficiency is *improved* by introducing disorder along the highway, and by imposing a speed limit.

Cellular automaton (CA) models have become increasingly important in the study of traffic flow. Although based on a very simple set of rules, these models are remarkable in their ability to simulate both gross and subtle features of real traffic. In one dimension, Nagel and Schrekenberg [1] introduced a stochastic discrete automaton model to study the transition from laminar traffic flow to start–stop-waves as the car density increases. Variations on the basic model include introducing separation-dependent car velocities [2], the addition of slower sites and takeover sites [3], studies of the effect of bottlenecks [4] and quenched disorder [5], etc. Similar models have been introduced for traffic flows in more than one lane [6], in crossings of one-dimensional (1D) lanes [7] and in two-dimensions [8] with alternate movements of eastbound and northbound cars simulating the effects of traffic lights. Effects of inhomogeneities, such as faulty traffic lights [9] and various mean-field theories [10], have also been studied in 2D.

In this paper we consider traffic flow along a long road, such as a highway, within the context of a one-dimensional (1D) cellular automaton (CA) array. We employ the Nagel–Schreckenberg (NS) model [1] and introduce two modifications: a rule which simulates the disparity between braking and acceleration, and a set of rules which model a junction along the highway. We begin with a 'bare' highway (i.e. no junction) in order to better understand the model's behaviour as the parameters are varied; various analytic results are given. We then introduce the junction and study the flow of cars through it as we vary these same parameters. We obtain the surprising result that the junction operates more smoothly when there is disorder on the road itself. Junction performance is also improved by limiting the speed of cars along the highway.

The basic 1D asymmetric exclusion model is defined on a lattice of length N, with N usually taken to be as large as is computationally convenient. Each site in the CA lattice has two possible states: 'occupied' by a car and 'vacant'. The rule for updating the state at

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each site for the most basic model is as follows: all vacant sites assume the state of sites to their immediate left, and all occupied sites assume the state of the site to their immediate right. This implies that cars move to the right if and only if there is a space to their right. A car may not move into an occupied site even if the occupying car is moving on in the same step.

The NS model involves two additional rules that produce a closer simulation of real traffic. (a) Cars may move with a range of integer speeds, $s = 0, \ldots, s_{\max}$. A car with speed $v = s - 1$ on the previous step will move in the next step with a speed given by the *lowest* of the following quantities: (i) $v = s$, (ii) $v = s_{\max}$, or (iii) v equal to the number of vacant sites to its immediate right. This will be referred to as the *acceleration* rule. (b) The cars are subject to a random disordering effect as follows. For each car whose scheduled speed for the next update is $v > 0$, there is a probability P_{fault} that it will in fact move with speed $v - 1$. This will be referred to as the *disorder* rule.

The second rule is intended to reflect the flawed behaviour of real (human) drivers. In this spirit, we will now introduce a further rule, referred to as the *slow-to-start* rule. Our rule will model the small, but finite delay before a car pulls away from being 'static', i.e. when it has reached the head of a queue. This can arise from a driver's loss of attention as a result of having been stuck in the queue, or from the slow pick-up of his vehicle's engine. This rather subtle feature of real traffic is likely to become important at high car densities, particularly since no such delay is likely to occur as cars decelerate, i.e. as they brake. The resulting asymmetry is liable to cause queues to lengthen. We define the slow-to-start rule as follows: *a given static car moves either on its first opportunity with probability 1 −* P_{slow} *or second opportunity with probability* P_{slow}. We note that the disorder rule can also cause cars to be slow in moving off from the heads of queues. However, the disorder rule affects vehicles of all velocities with equal probability; it introduces a general 'noise' into the system. By contrast, the slow-to-start rule affects only static cars on the first occasion that they are free to move; it reflects a distinct physical phenomenon of driver behaviour as described above.

In figure 1 we demonstrate the effect of the slow-to-start rule on traffic flow; in particular we contrast typical 'snap-shots' of the steady state with $P_{\text{slow}} = 0$ (left panel) and $P_{\text{slow}} = 0.5$ (right panel). It is clear that a qualitative change occurs in the distribution as a result of introducing a non-zero P_{slow}; the queues become less fragmented and the inter-queue regions widen. In fact, the two rules compete in this respect: the slow-to-start rule causes queues to merge while the disorder nucleates new queues. The mean length of queues in the steady state depends critically on the relative values of P_{slow} and P_{fault}. We shall see later that this interplay can have important consequences for highway junctions.

Figure 2 shows results for the flux f of cars as a function of the car density ρ. The flux is defined as the number of cars moving in a given step divided by the number of sites, and is therefore a measure of the highway's efficiency. The three flux-density relations obtained from the CA simulation correspond to the slow-to-start rule alone ('experiment'—long-dashed curve), the disorder rule alone ('experiment'—short-dashed curve), and both rules together ('experiment'—solid curve). Also shown are the analytical results from the theory presented below ('theory'—full circles). Note that theory and experiment are indistinguishable for the top two curves. These plots are for the case $s_{\max} = 1$, but the theory developed below is actually valid for all $s_{\max}$. The CA results are obtained from simulations on a chain of 1500 sites. A periodic boundary condition is assumed so that both the total number of cars and ρ are conserved. This is the usual boundary condition for traffic simulation, although 'open' CA models without conservation have also been studied [11]. For each initial configuration of cars, results are obtained by averaging over 1000 time steps

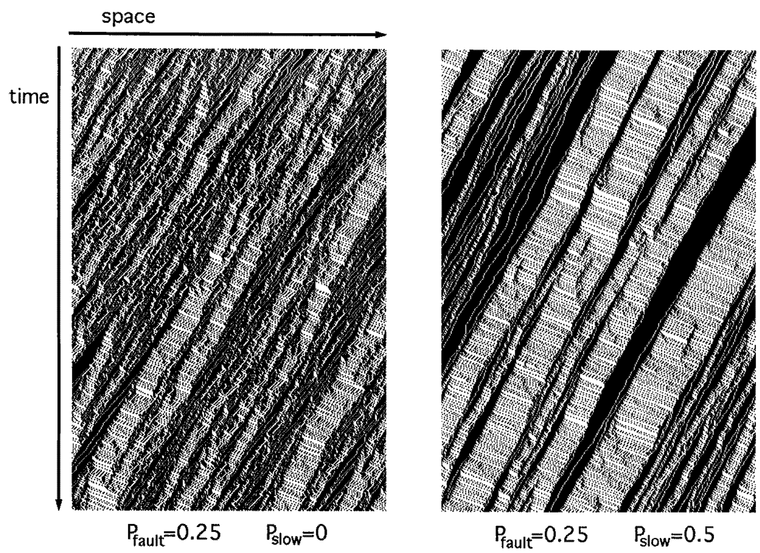

Figure 1. Each panel displays the distribution of cars along a highway over 500 consecutive time steps; a black pixel represents a car whilst a white pixel corresponds to an empty cell. The section of road is 400 cells wide. Left panel: $P_{\mathrm{fault}} = 0.25$, $P_{\mathrm{slow}} = 0$; right panel: $P_{\mathrm{fault}} = 0.25$, $P_{\mathrm{slow}} = 0.5$. Typical parameter values for a realistic highway are expected to lie between these two cases. For both panels $s_{\mathrm{max}} = 3$.

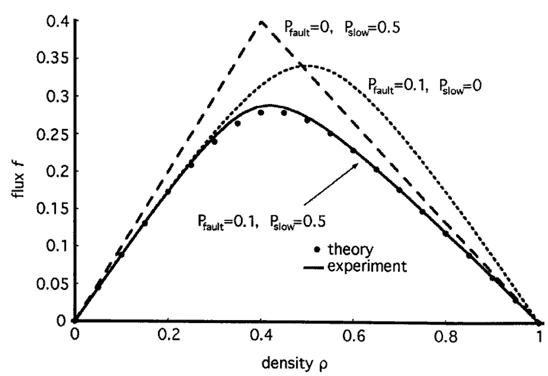

Figure 2. Flux-density relations (i.e. 'fundamental diagram') for various values of P_{fault} and P_{slow}. In all cases $s_{\mathrm{max}} = 1$. The CA simulation results are shown for the slow-to-start rule (long-dashed curve), the disorder rule (short-dashed curve) and the combination of the two rules (solid curve). Also shown are the analytic results (full circles). Note that for the upper two curves, the analytic results ('theory') and the CA simulation results ('experiment') are indistinguishable.

after the first 2000 steps, so that the long-time limit is approached. This criterion was found to be sufficient to guarantee a steady-state being reached. For each car density, results are averaged over 50 different initial configurations.

For the slow-to-start rule acting alone (the long-dashed curve in figure 2) we have chosen to set the parameter $P_{\mathrm{slow}} = 0.5$. The maximum flux occurs at $\rho = 0.4$. For $\rho > 0.4$, the flux decreases linearly with car density. These features can be understood by considering the system with a high car density (e.g. $\rho = 0.6$). There will always be substantial queues in this limit which do not contribute to the flux. We can estimate the flux if we know the density of cars in the free-flowing regions or, instead, the average number of sites n associated with each free moving car. This quantity n is determined by the rate at which cars leave the head of the queue bounding the free-flowing region on its immediate left. Consider the basic model ($P_{\mathrm{slow}} = 0$). A car will move off from the head of the queue on every time step; there will then be $(s_{\mathrm{max}} + 1)$ sites per car in the free flowing region†. With $P_{\mathrm{slow}} > 0$, some cars wait until the second possible time-step before moving from the head of the queue. Such a car will effectively occupy $(s_{\mathrm{max}} + 1) + s_{\mathrm{max}} = 2s_{\mathrm{max}} + 1$ sites, because of the wasted turn when the other free moving cars to its right all move s_{max} sites further away. The proportion of such slow cars is P_{slow}, so the average number of sites per free-moving car is $n = (s_{\mathrm{max}}+1)(1-P_{\mathrm{slow}}) + (2s_{\mathrm{max}}+1)P_{\mathrm{slow}} = 1 + s_{\mathrm{max}}(P_{\mathrm{slow}}+1)$. Let Y be the number of free-moving cars and X be the number of cars involved in queues. These two quantities are related to the total number of sites N and number of cars ρN by

$$\begin{aligned} N &= X + [1 + s_{\mathrm{max}}(1 + P_{\mathrm{slow}})]Y \\ \rho N &= X + Y. \end{aligned} \tag{1}$$

The flux is entirely due to the free moving cars and is given by

$$f \equiv \frac{Y}{N} = \frac{1}{(1 + P_{\mathrm{slow}})}(1 - \rho)\ . \tag{2}$$

This function is valid for sufficiently large ρ so as to produce queues in the steady state, i.e. all ρ for which $X > 0$. The transition‡ occurs at a density $\rho = \frac{1}{1+s_{\mathrm{max}}(1+P_{\mathrm{slow}})}$. Below this density, the absence of queues means that all cars are free flowing, and $f = s_{\mathrm{max}}\rho$. A complete description of the flux is

$$f = \begin{cases} s_{\mathrm{max}}\rho & \text{for } \rho < \dfrac{1}{1 + s_{\mathrm{max}}(1 + P_{\mathrm{slow}})} \\[2ex] \dfrac{1}{(1 + P_{\mathrm{slow}})}(1 - \rho) & \text{for } \rho > \dfrac{1}{1 + s_{\mathrm{max}}(1 + P_{\mathrm{slow}})}\ . \end{cases} \tag{3}$$

For $P_{\mathrm{slow}} = \frac{1}{2}$, the turning point should arise at $\rho = \frac{2}{5}$, after which the gradient should be $\frac{2}{3}$. This analytic result matches exactly with the simulation in figure 2, where the peak flux lies at $\rho = 0.4$, and the gradient in the region $\rho > 0.4$ is ≈ 0.65. Such good agreement has also been found for other values of P_{slow} and s_{max}. It is interesting to note that strictly in the limit of $P_{\mathrm{fault}} = 0$ and $s_{\mathrm{max}} = 1$, the action of the slow-to-start rule becomes identical to the *cruise-control* rule [13]. However, these two rules are *not* identical for $P_{\mathrm{fault}} > 0$ or $s_{\mathrm{max}} > 1$. We note that the expression in (3) reduces in the $P_{\mathrm{slow}} = 0$ limit to the expression

$$f = \begin{cases} s_{\mathrm{max}}\rho & \text{for } \rho < (s_{\mathrm{max}} + 1)^{-1} \\ (1 - \rho) & \text{for } \rho > (s_{\mathrm{max}} + 1)^{-1} \end{cases} \tag{4}$$

which has been obtained by Nagel and Herrmann [12] among others.

† Note that if $s_{\mathrm{max}} > 1$, this statement becomes an approximation since it neglects the small region at the head of a queue where cars are accelerating. However, the approximation is a good one since in this $P_{\mathrm{fault}} = 0$, $P_{\mathrm{slow}} > 0$ model the steady-state features a *small* number of widely-separated long queues.

‡ If ρ lies in the range $1/(1+s_{\mathrm{max}}(1+P_{\mathrm{slow}}))$ to $1/(s_{\mathrm{max}}+1)$, then a certain small sub-set of initial configurations will result in a steady-state with no traffic queues. The system therefore has two possible steady state solutions for any ρ in this region; however, the queue-less solution will tend to be 'washed out' when we average over many initial configurations; moreover, it will vanish entirely when we move to the more realistic $P_{\mathrm{fault}} > 0$ model.

For the disorder rule (short-dashed curve in figure 2) we have chosen to set the parameter $P_{\mathrm{fault}} = 0.1$. It turns out that one can derive an exact analytic expression for this $s_{\mathrm{max}} = 1$ system (see, e.g., [14]). The form is

$$f = \tfrac{1}{2}\left(1 - \sqrt{1 - 4(1 - P_{\mathrm{fault}})\rho(1-\rho)}\right) . \tag{5}$$

This may be obtained using an 'n-cluster expansion' in which one considers the probabilities $P(c_1, c_2, \ldots c_n)$ of finding a randomly selected string of n consecutive cells to be in states $c_1, c_2, \ldots c_n$. Whilst the above exact expression for $s_{\mathrm{max}} = 1$ can be obtained using just an $n = 2$ cluster treatment, it was found that in order to closely model systems with higher s_{max}, correspondingly larger clusters must be considered. This is understood to result from long-range correlations that exist in all systems with $s_{\mathrm{max}} > 1$.

The combined action of these two rules produces the 'fundamental diagram' shown as a solid curve in figure 2, for a system with $s_{\mathrm{max}} = 1$ and both P_{fault} and P_{slow} finite. Along this curve we display (full circles) the analytic results that we obtained from an $n = 2$ cluster treatment (see the appendix for an outline derivation). We can see that the fit, whilst good, is no longer exact. This is an indication that introducing the slow-to-start rule increases the distance over which correlations exist. In fact, this correlation can be traced to the lengthening of queues beyond the statistically expected length (cf figure 1).

We now turn to the highway containing a junction where cars may enter and leave. Two nearby, but non-adjacent, sites are chosen to be the 'input' and the 'output' sites, with the input site to the right of the output site so that cars entering must essentially traverse the entire road before exiting. Associated with the input site is an integer Q which is the number of cars queuing in the feeder road (or 'ramp') waiting to enter the highway. Cars are added periodically to the input queue ($Q \to Q + 1$); we choose a rate of one car added every five time steps. Whenever $Q > 0$ and the input site is vacant, this site becomes occupied and $Q \to Q - 1$. We delete one car entering the output site for every car added to the highway so that the total number of cars on the road is conserved, apart from small fluctuations in short time intervals between the addition and removal of cars. The quantity Q thus measures the flow of cars through the junction. It is desirable to keep Q low; indeed real junctions may only be able to support a finite number of waiting cars before becoming catastrophically locked up.

Figure 3 shows $\bar{Q}$, the value of Q averaged over the last 2000 of 4000 steps, as a function of the disorder probability P_{fault} for different values of s_{max} and P_{slow}. A car density of $\rho = 0.5$ is chosen for all simulations. The lowest three curves correspond to $s_{\mathrm{max}} = 1$ and $P_{\mathrm{slow}} = 0, 0.25, 0.5$, respectively. With $P_{\mathrm{slow}} = 0$, $\bar{Q}$ is small and increases slightly with P_{fault}. This is expected since in the steady state of the corresponding junctionless model, every other site is empty in the $P_{\mathrm{fault}} \to 0$ limit. The introduction of a single junction does not significantly alter this distribution, so cars can easily filter onto the road and $Q(t)$ remains small for all t. If we increase P_{slow} while setting $P_{\mathrm{fault}} = 0$, $Q(t)$ behaves very differently, occasionally flaring up to large values. This is shown in the left inset, in which the number of dots in a column represents the value of Q, and each successive column is advanced by five time steps. The typical value of the maximum does of course depend on the parameters such as ρ, P_{slow}, and the rate at which cars are added to the queue in the feeder road; however, the appearance of this feature is quite general. In order to understand the phenomenon we examine the spatial distribution of cars in the steady state. We find that the slow-to-start rule, with or without the junction, gives a steady state with fewer but longer queues relative to the basic model. When one of these queues, which move backwards along the road, passes a junction, it prevents cars from entering the road for a substantial period of time. It is during this time that the value of Q increases dramatically.

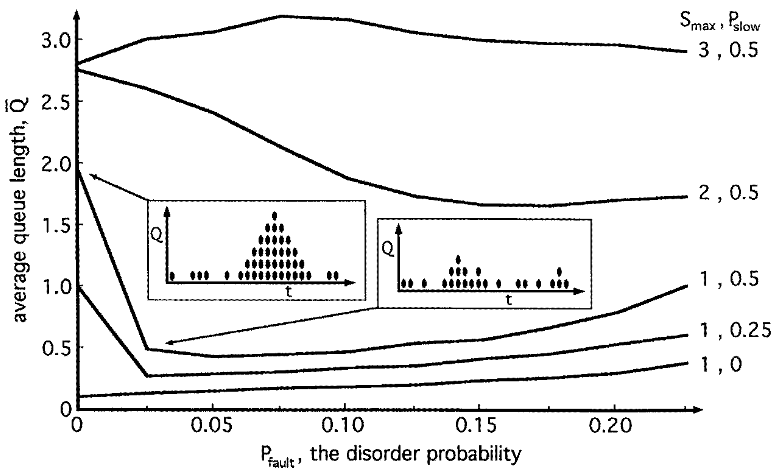

Figure 3. Average length of the feeder-road queue $\bar{Q}$ as a function of P_{fault} (disorder rule) for various values of s_{max} (acceleration rule) and P_{slow} (slow-to-start rule). Insets: the number of dots in each column represents Q at a given time-step and each successive column represents an advance of five time-steps.

It is worth spending a moment looking into why such large queues form on roads (with or without junctions) employing the slow-to-start rule alone (no disorder, i.e. $P_{\mathrm{fault}} = 0$). The explanation lies not in the fact that the cars are slow to move off, but rather in the uncertainty in this delay, which allows the queue lengths to vary. It is possible for a queue to shrink to zero, i.e. evaporate completely, but there is no corresponding mechanism allowing for creation of queues. Since the total number of queued cars remains approximately constant, it is clear that the average length will increase. The ultimate limit for a closed system is that all queues merge into one; however, it would take an astronomical time to reach this state on a long road.

If we now allow both P_{fault} and P_{slow} to be non-zero, we make the interesting observation that *disorder* can *improve* the junction's performance. The third curve in figure 3 drops dramatically as P_{fault} is increased from zero. The value of $\bar{Q}$ is reduced from 1.9 for $P_{\mathrm{fault}} = 0$ to 0.5 for $P_{\mathrm{fault}} = 0.025$. In the corresponding inset, we see that the value of Q no longer flares up. The disorder rule breaks up the long queues resulting from the slow-to-start rule alone by increasing the *number* of queues, without significantly altering the car density in the inter-queue regions. Each momentary driving defect has a chance of nucleating a queue. As the average length of the queues on the road decreases, the maximum number of cars waiting to enter also decreases.

The surprising conclusion from the model is that it is 'beneficial' to create queues on the highway. The queues effectively compete with one another for the static cars, the total number of which remains practically constant over time. When a queue becomes deprived of static cars it is destroyed, so without the introduction of new queues the road becomes highly inhomogeneous: only a small number of large queues survive. This inhomogeneity has a markedly detrimental effect on road junctions, which seize up when one of the large queues moves past.

Finally we turn to the upper three curves in figure 3, which between them display the effect of increasing s_{max} at a fixed value of $P_{\mathrm{slow}} = 0.5$. We observe that with increasing

s_{max}, (a) $\bar{Q}$ increases, and (b) the beneficial effect of increasing P_{fault} diminishes. To understand observation (a), we note that the queue lengths along the road (which directly affect $\bar{Q}$) increase with the choice of s_{max}; in fact this occurs for any given values of P_{fault} and P_{slow}, so we may take the limit $P_{fault} = P_{slow} = 0$ to understand the effect. Since for $\rho \geqslant 0.5$ the flux is independent of s_{max} (see equation (4)) it is clear that as s_{max} increases, the cars travelling at high speed must be counter-balanced by a greater number stuck in queues. Observation (b) is due to the inability of the disorder rule to nucleate queues when it acts on fast-moving cars. For a queue to form spontaneously, a car must be made stationary. However, the disorder rule only reduces the velocity of cars by one unit, so that the probability of actually halting a car which initially moves with speed s_{max} (by the disorder rule, this would imply decelerating it on s_{max} consecutive time-steps) falls approximately as P_{fault} to the power of s_{max}. In real traffic situations, it is indeed the case that jams do not tend to form spontaneously in regions of a road where cars are moving very quickly.

In conclusion, we have studied the performance of a junction under the action of three different rules. The acceleration and disorder rules due to Nagel and Schrekenberg were employed, and we introduced a third *slow-to-start* rule which reflects a feature of real driving that is distinct from general disorder. Having quoted and obtained analytic forms describing the effects of these rules on a bare road, we then applied them to the junction. We measured the junction's performance by the variable Q, which is the length of the queue of cars forced to wait in the 'feeder-road' or 'ramp'. We studied in detail the effects when the road is half-filled with cars, i.e. $\rho = 0.5$. Our main findings, for which we have provided qualitative explanations, are as follows: (i) when the cars on a highway are constrained to move slowly (i.e. $s_{max} = 1$) the junction's performance is maximized by introducing a finite degree of disorder along the road. (ii) As the speed limit is relaxed, i.e. for larger s_{max}, we note that (a) the performance of the junction is reduced, and (b) the beneficial effect of disorder diminishes. Noting that for $\rho \geqslant 0.5$ the flux along the road is not significantly altered by the choice of s_{max}, we may conclude that *it is desirable to set a speed limit near junctions on busy single-lane roads*.

The systems we have studied were designed to be both plausible and intuitive, and yet still permit a certain degree of analytical analysis. We believe that the general characteristics of our model are indeed consistent with personal experience. In order to establish the extent to which this (or any) CA traffic model makes accurate *quantitative* predictions about real traffic flow, one should clearly make a thorough comparison with empirical traffic data for the same highway/junction system, if available. Such a comparison lies beyond the intended scope of this paper.

Acknowledgments

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Appendix

Here we outline the method for applying the cluster expansion [14] to our model with $s_{max} = 1$ and finite P_{slow}. When we consider the state of the array just after movement, we see that there are four states that a cell may be in: car moving with velocity 1 (denoted '1'); car static due to the action of the slow-to-start or disorder rules (denoted 's'); car static due to blockage ahead (denoted 'b'); and an empty cell (denoted 'e'). Following [14], we can reduce the problem to three possible states by considering the road at an intermediate stage between one movement and the next. Consider that a single time-step consists of the following stages: (i) acceleration—all cars are assigned a velocity of '1'; (ii) slow-to-start—all cars that are legitimate candidates may be decelerated to 's' with probability P_{slow}; (iii) blockage—all cars which are blocked from moving have their state changed from '1' to 'b'; (iv) disorder—all cars still in state '1' may be decelerated to 's' with probability P_{fault}; (v) movement—all cars in state '1' are moved one cell to the right. Now consider the state of the road after the action of the acceleration and slow-to-start stages, but before the action of the blockage stage. In this way we avoid having to consider cells in state 'b'. When we come to the expression for the flux we must apply the remaining rules, i.e. consider only $P(10)$ and apply the factor $(1 - P_{\mathrm{fault}})$. Recall that $P(c_1, c_2)$ is the probability of finding a randomly selected string of two consecutive cells with states c_1 and c_2 respectively. In the present work we consider only a two-cluster expansion; this was found to be sufficient to model the corresponding fundamental diagram to within 2% accuracy (see figure 2). The two-cluster probabilities obey the following identities at all times:

$$
\begin{aligned}
P(S1) = P(SS) = 0 \qquad & P(S0) = P(1S) + P(0S) \\
& P(01) = P(10) + P(1S) \\
& \rho = P(01) + P(11) + P(S0) \\
& 1 - \rho = P(00) + P(10) + P(S0)\,.
\end{aligned}
\tag{A1}
$$

With the application of these identities we are left requiring three equations. We employ the following, which are approximately true in the equilibrium limit:

$$
\begin{aligned}
P(S0) &= q(1-s)\,(P(1110) + P(0110)) \\
P(11) &= \epsilon\,(qP(1011) + P(1111) + P(0111)) + P(11S0) + P(111S) \\
&\quad + qP(10S0) + P(01S0) + P(011S) + pP(1110) + p(P0110) \\
&\quad + q(P101S) + qpP(1010) \\
P(10) &= pP(0100) + qsP(0110) + qP(1000) + q^2P(1010) + P(1S00) \\
&\quad + P(0S00) + qP(1001) + qsP(1110) + pP(0101) + pP(1100) \\
&\quad + pP(1101) + P(1S01) + P(0S01) + qP(100S) + pP(010S) \\
&\quad + pP(110S) + P(0S0S) + P(1S0S)\,.
\end{aligned}
\tag{A2}
$$

For compactness we have used

$$
p = P_{\mathrm{fault}} \qquad q = (1 - P_{\mathrm{fault}}) \qquad s = 1 - P_{\mathrm{slow}}\,. \tag{A3}
$$

The quantity $P(WXYZ)$ is of course expanded in the two-cluster expansion as

$$
P(WXYZ) = P(W|\bar{X})P(XY)P(\bar{Y}|Z) \tag{A4}
$$

where the conditional probabilities

$$
P(W|\bar{X}) = \frac{P(WX)}{\sum_i P(Wi)} \qquad P(\bar{Y}|Z) = \frac{P(YZ)}{\sum_i P(iZ)}\,. \tag{A5}
$$

Note the important correction ϵ in the expression for $P(11)$:

$$
\epsilon = (1 - qpP(\bar{1}|1))\,. \tag{A6}
$$

The seven equations contained in expressions (A2) and (A3) are solved simultaneously to find the quantity $P(10)$ in terms of the constants ρ, P_{fault}, P_{slow}. The flux then follows by $f = qP(10)$.

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